# Monetary Lotteries

- Let us now consider lotteries over money
- Let a continuous variable *x* denote the amount of money
- Let *F*(*x*) denote a cumulative distribution function (cdf) over *x*.
- The (v.N-M) expected utility function takes values:

$$U(F) = \int u(x)dF(x)$$

- where *u*(*x*) the utility of getting the amount *x* with certainty (probability=1) and is called Bernoulli

# Risk aversion and concavity

- A decision maker is a risk averter (is risk-averse) if for any lottery $F(x)$ the degenerate lottery which yields the expected (average) payoff $\int x dF(x)$ with certainty is at least as good as the lottery $F(x)$, i.e.

$$U(F) = \int u(x)dF(x) \leq u\left(\int xdF(x)\right)$$

- The above is called *Jensen's inequality,* and it is one of the definitions of a concave function ($u(x)$).
- Risk aversion is therefore equivalent to concavity of the Bernoulli utility function

# Certainty Equivalent

- *Certainty equivalent* of $F(x)$ is denoted by $c(F, u)$ is such amount of money, that the individual is indifferent between getting that amount with certainty or getting the lottery $F(x)$, i.e.

$$u(c(F,u)) = \int u(x)dF(x) = U(F)$$

- graph

# Probability Premium

- For **any** fixed amount of money *x* and a positive number *ε*, the *probability premium,* denoted by π(*x*, *ε*, *u*) is the excess (over fair odds) in the probability of winning, that makes the individual indifferent between the certain outcome *x* and a gamble between *x* + *ε* and *x* – *ε,* i.e. such that

$$u(x) = (\tfrac{1}{2} + \pi(x,\varepsilon,u))u(x+\varepsilon) + (\tfrac{1}{2} - \pi(x,\varepsilon,u))u(x-\varepsilon)$$

- graph

# Equivalence theorem

- The following properties are equivalent:
  - The decision maker is risk-averse
  - $u(x)$ is concave
  - $c(F, u) \leq \int x dF(x)$ for all $F(x)$
  - $\pi(x, \varepsilon, u) \geq 0$ for all $x$ and $\varepsilon$

# Arrow-Pratt coefficient

- The Arrow-Pratt coefficient of **absolute** risk aversion at point *x* is defined as $r_A(x) = -u''(x)/u'(x)$.
- Notice that A-P coefficient is in fact a transformation of the utility function, that preserves all important information

# Comparisons across individuals

- An individual, who's preferences can be described by a Bernoulli utility function $u_2(.)$ is more risk averse than an individual with $u_1(.)$ if (all definitions below are equivalent):
  - $r_A(x, u_2) \geq r_A(x, u_1)$
  - $c(F, u_2) \leq c(F, u_1)$
  - $\pi(x, \varepsilon, u_2) \geq \pi(x, \varepsilon, u_1)$
  - there exists a concave function $\varphi(.)$ such that $u_2(x) = \varphi(u_1(x))$, i.e. $u_2(.)$ is „more concave" than $u_1(.)$
  - whenever a lottery $F(.)$ is preferred to a reckless outcome (sure amount) $x^*$ according to $u_2(.)$, it is also preferred to $x^*$ according to $u_1(.)$

# Coefficient of relative risk aversion

- The coefficient of **relative** risk aversion at point *x* is defined as
$r_R(x) = -\, x u''(x)/u'(x)$.
- Decreasing relative risk aversion means that as wealth increases, the individual becomes less risk averse with respect to gambles that are the same *in proportion* to his wealth.
- Decreasing relative risk aversion implies decreasing absolute risk aversion, i.e. as wealth increases, the individual becomes less risk averse with respect to gambles that are the same *in absolute value*.
- Finance theory often assumes constant relative risk aversion.